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The allocation of entrepreneurial effort and its implications on economic growth *

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Abstract: The problem to allocate effort to innovation activities is defined and modelled for any single entrepreneur according to its propensity to innovate, which combines pure innovation and rent-seeking strategies. The allocation problem is solved both analytically and via simulation. The individual decisions measured in units of innovation are then aggregated to calculate the innovation quantity for a given population based on the distribution of heterogeneous entrepreneurs. The entrepreneurship rate and the implications for economic growth are also quantified. Consequently, policy makers should focus on reducing the entry barriers and the costs of production in order to stimulate the entrepreneurial activity and maximize the innovation quantity. They should also foster the attitude and propensity towards innovation.

Keywords: entrepreneurial heterogeneity, propensity to innovate, endogenous growth

JEL: O12, M13, O40

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1. Introduction

The main contribution of the entrepreneur within the economy, in terms of rate of growth, consists of increasing the productivity of the system through innovation. The immediate consequence of innovation is the introduction of novelties within the productive process, new types of more productive capital goods, or consumption goods with greater added value. The maximum potential growth would be then associated with the use of the greatest part of the existing entrepreneurial capacity and, consequentially, of available resources in innovative activities. On the other hand, the allocation of entrepreneurial capacity to rent-seeking (non-productive) activities generates a lower economic performance in terms of *per capita* output growth and, as a limit case, may degenerate into a contractive economy (Murphy et al., 1991, 1993).

Entrepreneurial decisions in this context refer to the allocation and coordination of scarce resources between innovation activities and rent seeking. Entrepreneurs will have to direct their effort towards the combination of activities that maximize their innovation utility, which will in turn potentiate economic growth. Policy makers should then understand how their policies affect the entrepreneurial decisions and their impact on the rate of economic growth.

As for the first extreme type of entrepreneurs, pure innovators will in general perceive differently the costs and benefits as well as the risks of innovation. The reason is that his “vital objective” relates to the introduction of innovations, and not just the maximization of economic profits (see Keynes, 1936: Chap. XII). In the case of this pure innovative entrepreneur, the type of adopted critical judgment is most likely related with a spontaneous impulse for economic change.

However, entrepreneurs may behave as rent-seekers, the second extreme type: their activity consists mainly of pursuing the maximum possible profits not linked to innovations –or simply to activities that generate productivity gains. By behaving in such a manner, depending on their capacity to absorb resources of the economy, this type of entrepreneurship may end up rationing scarce resources required by the innovator-entrepreneurs to develop their activities; i.e.: rationing the necessary resources that will truly promote increases in productivity of the economy in the long term.

Departing from Baumol (1968) seminal work, we develop a quite simple idea in order to link different entrepreneurial activities under a single “entrepreneurial problem”. We do so by introducing entrepreneurial heterogeneity by assuming that entrepreneurial activities may be any combination of the two extreme kinds: innovative and rent-seeking. However, contrary to Baumol’s claim that “no exhaustive analysis of the process of allocation of entrepreneurial activity among the set of available options will be attempted” and therefore “only at least one of the prime determinants of entrepreneurial behavior at any particular time and place is the prevailing rules of the game that govern the payoff of one entrepreneurial activity relative to another” (Baumol, 1990: 898, emphasis in the original), we model different entrepreneurial

activities without assuming that entrepreneurs are *essentially* different, and explore the implications of different structures of payoffs over innovation activities.

As a consequence, an attitude towards innovation is introduced and distributed among a (theoretical) population of entrepreneurs.¹ Thus, we define an *entrepreneurial distribution function* between the two extremes –pure innovators and rent-seekers–, a function that captures the *hypothesis of heterogeneity* of entrepreneurs. The distribution of entrepreneurs’ attitude towards innovation or *propensity to innovate* consists of assuming that each entrepreneur in the economy establishes the relative value of each of the two extreme types together with the restrictions that impinge upon these values.

This article assumes that each type of entrepreneur maximizes its *utility* function associated to innovation opportunities and the possibility of obtaining economic rents, subject to a minimum level of (subjective) profits. Given its attitude towards innovation, and depending on the availability of alternative earnings within the economic system (rents), an entrepreneur decides how many resources they will devote to innovation, taking also the entry barrier costs into account. Formally, the entrepreneurial problem (EP) could be then stated as follows for each entrepreneur:

$$\begin{array}{ll} \text{Max} & \text{Utility due to entrepreneurial effort} \\ \text{s.t.} & \text{profit is at least enough to compensate entry barrier costs and alternative} \\ & \text{sources of gains} \end{array}$$

We then embed this individual behavior into an endogenous growth model in order to understand the process of “creating entrepreneurs” and its implication in the economy. The individual innovation efforts are aggregated for a given distribution of entrepreneurs and the overall quantity is included into the growth model to derive police implications. The rate of entrepreneurship is also calculated as a by-product.

The structure of the paper is as follows. Section 2 presents the model of entrepreneurial heterogeneity and poses the choice that entrepreneurs face -the (EP) problem. Section 3 investigates the possible decisions that entrepreneurs might take on how to allocate their efforts depending on their attitude towards innovation. Section 4 links together entrepreneurship and economic growth, and Section 5 provides a numerical illustration of our argument by means of simulation applied to the simplest case of a linear model. The paper finishes with some concluding remarks and policy implications of the model.

2. The heterogeneous entrepreneur allocation problem

In a letter to F.A. Walker, Léon Walras recognised that the definition of entrepreneur was, under his point of view, “*le nœud de toute l’économique*” (Walras, 1965). Indeed, this concept plays a fundamental role in the explanation of many economic facts —like economic growth—, but at the same time there must exist few central economic theory concepts that have been understood and applied in such a diverse manner (Casson, 1982). In Baumol’s famous words, “the Prince of Denmark has been expunged from the discussion of *Hamlet*” (Baumol, 1968: 66).

¹ Heterogeneity is represented by means of an entrepreneurial density function in which entrepreneurial attitude towards innovation ranges between pure innovator and pure rent-seeker.

In some economic theories the entrepreneur is the agent who, in the search for their own profit, coordinates the markets (Walrasian and neoclassical economics and, in a very especial sense, Austrian economics (Kirzner, 1992). In the case of the Schumpeterian entrepreneur (Schumpeter, 1932 [2005], 1934) the entrepreneur is the agent who has the will to introduce new combinations in the field of production. And finally, there are entrepreneurs whose very special nature leads them to assume the risks involved in all the decisions that are taken in contexts of true uncertainty (Knight, 1921).

Our first aim is to first characterize this multidimensional economic agent and, then, define and solve its innovation allocation problem in terms of:

- the decision variable: innovation quantity
- the objective function: maximize utility gains
- the necessary condition to innovate: obtain enough profit to compensate at least the entry barrier costs

Characterization of entrepreneurship

Traditionally, the innovation activity -the workhorse of economic growth- is associated with the hypothesis that entrepreneurs just behave as profit maximizers. However, rent-seeking may be considered also as an entrepreneurial activity; in this case, the explanatory power of this behavioral assumption is limited. Rent-seeking behavior has a long history in economics, dating back to the seminal work of Tullock (1967). The basic idea is best demonstrated explaining the social welfare losses involved in the establishment of monopolies, tariffs, and subsidies (Tullock, 1987). There are recent attempts to propose models of rent-seeking behavior for explaining the resource curse phenomenon -see Torvik (2002), Mehlum, et al. (2006), Hodler (2006), Arezki and Brückner (2010). Another approach consists of examining the political economy of intellectual property, private and public rent-seeking and its role for the social usefulness of innovation (Boldrin and Levine, 2004, 2008).

To characterize each entrepreneur, we describe the two extreme types of attitudes towards innovation that frame the strategies of the individuals:

- (a) type 0, entrepreneurs associated with pure innovative activities
- (b) type 1, entrepreneurs whose underlying (and unique) objective is the search for the maximum possible source of earnings –rents- in the economy.

By doing so, each entrepreneur might be represented by his specific position between the two extremes: his relative position is defined by $\delta \in [0,1]$. Therefore, by construction, δ is both the propensity and the “psychological distance” of any given entrepreneur from the extreme type 0 –being an innovator. Consequentially, $(1 - \delta)$ is the “psychological distance” from type 1 –being a rent-seeker. Each entrepreneur has therefore two ways of obtaining *utility gains*: via pure innovation; and via economic profit, which may be obtained either by innovating or by other alternative activities such as rent-seeking. How each entrepreneur weighs exactly each possibility of improving his personal situation is described by his propensity to innovate, δ .

The decision variable: the allocation of entrepreneurial effort

Each entrepreneur δ must first decide how much of the innovative good he would produce, should he choose to produce it at all, and then he must decide which activity he will finally opt for. Let $q(\delta)$ be the amount of innovation that an entrepreneur of type δ will produce under the assumption that an invention is available.² In other words, $q(\delta)$ is the effort towards innovation that an individual entrepreneur allocates.

Whenever facing the allocation problem, the entrepreneur will then have to decide between:

- (a) producing a positive quantity of the innovative good, $q(\delta) > 0$
- (b) obtaining a “rent” by carrying out other activities in some other sector of the economy; in this case $q(\delta) = 0$.

The objective function: utility gains

To decide on how to allocate the innovation effort among the different alternatives, the objective of a given entrepreneur δ is defined in terms of a *subjective utility function* that measures the degree in which the entrepreneur takes part in activities of type 0 and of type 1.

$$U(\delta, q) = (1 - \delta) u(q) + \delta v(\pi(q)) \quad (1)$$

The first term, $u(q)$, represents the *average earnings in utility terms* that the entrepreneur can obtain by innovating, which depends directly on the amount of the innovative good that is introduced into the economy. On the other hand, the second term, $\pi(q)$, is the profit associated with a given amount of the innovative good q and $v(\pi(q))$ is the *utility gains that this profit produces*. It is this second term that allows entrepreneur δ to compare the profits associated with this activity with the profit that can be obtained by alternative uses of his entrepreneurial capacity.³

The economic profit associated with the quantity of innovative good produced, q , is defined by:

$$\pi(q) \equiv (p(q) - c)q \quad (2)$$

where $p(q)$ is the demand function for the innovative good; and c is the average cost associated with the production of an additional unit of q .

The binding restriction: the necessary condition to innovate

Although maximizing profits, no entrepreneur would want to become bankrupt by carrying out the innovative activity that he values most. He will always search for a satisficing earning that is at least equal to the cost of innovating, α_0 , should he decide to repeat his plan in the future. The entrepreneur will then compare this earning, from his

² This amount may refer to a new investment good (capital) (Romer, 1990) or to an existing one but with new features that makes it more productive (Grossman and Helpman, 1991).

³ Note that the first term always refers to the utility obtained from innovating.

position δ , with his other options, the non-innovation rents, denoted by R . This minimal condition to innovate may be defined as follows:

$$\left[(p(q) - c)q = \pi(q) \right] \geq [\alpha(\cdot) = \alpha(\delta, \alpha_0, R)] \quad (3)$$

where $\alpha(\cdot)$ is a function that sets the minimum yield (satisficing threshold) that the plan of each type δ entrepreneur must obtain. This function $\alpha(\cdot)$ has as arguments:

- (1) the propensity of the entrepreneur towards innovation, δ
- (2) the cost of introducing an innovation or entry barrier, α_0
- (3) the gains that can be obtained in other sectors of the economy, R . (This last variable together with $\pi(q)$ represent the structure of payoffs of the economy.)

In order to innovate, that is, for $q(\delta) > 0$, it must happen that, the minimum condition $\pi(q) \geq \alpha(\cdot)$ is satisfied. If it is not satisfied for the corresponding values of α_0 and R , the entrepreneur chooses not to innovate in new goods, $q(\delta) = 0$, and engages in rent-seeking activities.

The entrepreneur's problem (EP)

We may then formally set the “allocation of entrepreneurial effort” problem as follows after consistently combining type of entrepreneur, decision variable, objective function and binding restriction:

$$(EP): \begin{cases} \max_{q \geq 0} U(\delta, q) = (1 - \delta)u(q) + \delta v(\pi(q)) \\ \text{s.t.: } \pi(q) \equiv (p(q) - c)q \geq \alpha(\delta, \alpha_0, R) \end{cases} \quad (4)$$

3. The decision on the allocation of entrepreneurial effort

The decision that each economic agent should take concerning the allocation of the entrepreneurial effort depends therefore on α_0 , R , c , p , and δ ;⁴ although the (EP) solution for each particular entrepreneur δ exists and it is unique.⁵ We denote this optimum innovation quantity by:⁶

$$q^*(\delta) = q(\alpha_0, R, c, p, \delta) \quad (5)$$

⁴ For simplicity, we have set α_0 and R as constants and equal for all entrepreneurs, identical for all types of innovations. The same assumptions can be applied to the average costs of production of an additional unit of the new good, c .

⁵ The continuity of the utility function and the fact that the feasible set is closed and bounded guarantees that there exists a global maximum (Weierstrass Theorem). Moreover, since the function is strictly concave and the feasible set is convex, the Fundamental Theorem of Convex Programming guarantees that the global maximum of (EP) is unique.

⁶ The different regimes of solution are summarized in Table 1 and their mathematical derivation is included in Appendix I.

For any solution regime, if any of the influential parameters are separately evaluated for sensitivity, it can be shown that:⁷

$$\frac{\partial q}{\partial \alpha_0} < 0; \frac{\partial q}{\partial R} < 0; \frac{\partial q}{\partial c} < 0; \frac{\partial q}{\partial p} > 0; \frac{\partial q}{\partial \delta} < 0; \quad (6)$$

In words, the quantity of innovation goods generated by the solution of (*EP*) is higher:

- (1) the lower the minimum profit claimed by entrepreneurs in innovative activities, α_0 ;
- (2) the lower the alternative sources of gains, R ;
- (3) the lower the cost of producing innovations, c ;
- (4) the higher the price of innovations, p ;
- (5) the higher the individual propensity to innovate, that is, the lower δ .

These statements look intuitive if taken one-at-a-time. However, if all the effect are jointly considered and analyzed, we may summarize the decision possibilities faced by the entrepreneur as follows: (a) there will never be innovation if the entry barrier cost is higher than the profit, regardless of the entrepreneurial type and the propensity to innovate; (b) there will always be innovation if the profit is higher than the alternative rent, regardless of the entrepreneurial type; and (c) otherwise, some entrepreneurs might innovate, depending on where the value of the profit lies with respect to the entry barrier and the rents.

4. Entrepreneurship and economic growth: some policy implications

The aggregate behavior of the economy depends on how, not the individual entrepreneurs allocate their efforts according to the (*EP*), but all of the entrepreneurs jointly carry out their activities. Different social structures of payoffs (Baumol, 1994) will result in different economic performances (in terms of growth of output, for example) through the different ways to allocate the efforts of innovation of the entrepreneurs. The internal evolution of the social dynamics also determines the distribution of entrepreneurs within a given economy between the two extreme types.

Modern endogenous growth models in economy stress the deep relationship that exists between entrepreneurship and the economic performance of a society as measured for example by the *per capita* output growth (Acemoglu, 2009). The answers provided by economic growth theory are rather varied, ranging from models that abstract entrepreneurial activity (see for example Mankiw, et al., 1992, Solow, 1956) to those that formally integrate entrepreneurship into models that incorporate the intentional actions of entrepreneurs in the explanation of how the rate of growth is determined (Aghion and Howitt, 1992; Aghion and Howitt, 1998; Romer, 1994).

We have decided to follow this second route due to the easy and consistent fit of the (*EP*) philosophy into the complex mathematics of the growth models, with the necessary

⁷ See Appendix III.

previous aggregation of individual decisions into macroeconomic magnitudes: aggregated innovation quantities and rate of entrepreneurship.

Aggregate innovation effort

The aggregate value of innovation for the economy as a whole, Q , is determined by adding the individual quantities of innovation across all the entrepreneurs δ . Thus:

$$Q = \int_0^1 q^*(\delta) f(\delta) d\delta \quad (7)$$

Let us denote by $f(\delta)$ the density function of the propensity to innovate, in order to characterize any given economy. A higher propensity of society towards innovation is captured in the model by a “higher” density of entrepreneur propensity to innovate. The shape of this last “variable” $f(\delta)$ may differ drastically between economies –both between different contemporaneous economies and the same economy at different times.⁸

The economic policy efforts should then be based on the maximization of not only the individual entrepreneurship activities, q^* , but also on the distribution of the propensities $f(\delta)$ in order to increase the number of entrepreneurs; that is, politicians should foster that entrepreneurs shift their propensity towards pure innovation $(1-\delta)$, lowering their individual δ .

Entrepreneurship rate

From the solution of (EP) it is also possible to directly define and calculate an index that refers to the rate of innovative entrepreneurs of a given economy. We define the *entrepreneurship rate* as δ_0 . Its value, which ranges between 0 and 1, is calculated by solving:

$$\pi_{max} = \alpha(\alpha_0, R, \delta_0) \quad (8)$$

Those entrepreneurs with $0 \leq \delta \leq \delta_0$ will allocate efforts to entrepreneurial activities since their propensity to innovate δ is close enough to pure innovation ($\delta=0$), while those with higher values of δ will not as they are closer to rent-seeking ($\delta=1$). The economic implications of this index are that policies centered in economic growth should strive for lowering both the entry barriers and the other rents of the economy and to increase the profit associated to innovation.

Economic growth

In order to study how the aggregate allocation of the entrepreneurial efforts affects the dynamics of the economy, we use an adapted version of Rivera-Batiz & Romer (1991) endogenous growth model for evaluating the evolution of the rate of change of

⁸ The main results are summarized in a formal proposition in Appendix I and the proofs are included in Appendixes III and IV.

innovations over time, denoted by γ_y . According to this model, the rate of change of innovations is:

$$\gamma_y = \frac{(1-\mu)\beta Q - \dot{Q}c}{QC - \alpha_0} \quad (9)$$

where we assume a consumption function $C = \mu Y$ with a constant marginal propensity to consume $0 < \mu < 1$; and β is the productivity of the innovation.⁹

This rate of change is increasing in Q and decreasing in \dot{Q} . That is to say, the rate of growth of the number of new goods is directly proportional to investment in the introduction of new goods — and the proportion is the productivity of new goods —, and is inversely proportional to the cost of developing and producing them, and in the production of existing goods. Once again, entry barriers and costs should be lowered, whereas productivity should be increased.

These statements perfectly fit into the framework of the individual (EP), giving consistency to the overall approach of this research that relates the allocation of innovation effort by heterogeneous entrepreneurs and the growth of the economy.

5. A numerical illustration

The (EP) model is both solvable analytically and by means of simulation techniques. Next we propose to solve a theoretical example. For the sake of simplicity, let us use linear functions and the following input values:

- (a) $p(q) = a - bq$, with $a, b > 0$
- (b) $u(q) = dq$, $d > 0$
- (c) $v(\pi(q)) = q$
- (d) $\alpha(\delta, \alpha_0, R) = (1-\delta)\alpha_0 + \delta R$

The formulation of the (EP) for each entrepreneur is then the following:

$$(EP): \begin{cases} \max_{q \geq 0} U(\delta, q) = du(q) + \delta\pi(q) \\ \text{s.t.: } \pi(q) \geq \alpha(\delta, \alpha_0, R) = (1-\delta)\alpha_0 + \delta R \end{cases} \quad (11)$$

Analytically, the solution is given by:

$$q^*(\delta, \alpha_0, R) = \begin{cases} \min\{q_1(\delta, R), q_2(\delta, R)\} & \text{if } \alpha(\delta, R) \leq \frac{(a-c)^2}{4b} \\ 0 & \text{if } \alpha(\delta, R) > \frac{(a-c)^2}{4b} \end{cases} \quad (12)$$

⁹ For more details, see Appendix V.

Where

$$\begin{aligned} q_1 &= \frac{(a-c)}{2b} + \frac{\sqrt{(a-c)^2 - 4b\alpha(\bullet)}}{2b} \\ q_2 &= \frac{(a-c-d)}{2b} + \frac{d}{2b\delta} \end{aligned} \quad (13)$$

The entrepreneurship rate (δ_0) follows:

$$\delta_0 = \frac{\frac{(a-c)^2}{4b} - \alpha_0}{R - \alpha_0} \quad (14)$$

We proceed to further assign numerical values to the input variables:

- $R = 0.33$
- $\alpha_0 = 0.01$
- $a = 5; b = 6.67; \text{ so } p(q) = 5 - 6.67q$
- $c = 2.75$
- $d = 0.20; u(q) = 0.20q$

Figure 1 shows the quantity of innovation for each entrepreneur as a function of the distance δ to pure innovation. The theoretical break point or entrepreneurship rate is at $\delta_0 = 0.5617$ for any $f(\delta)$. It also shows that the quantity is calculated from function q_1 if $\delta < 0.091$ and from function q_2 if $0.091 < \delta < 0.5617$.

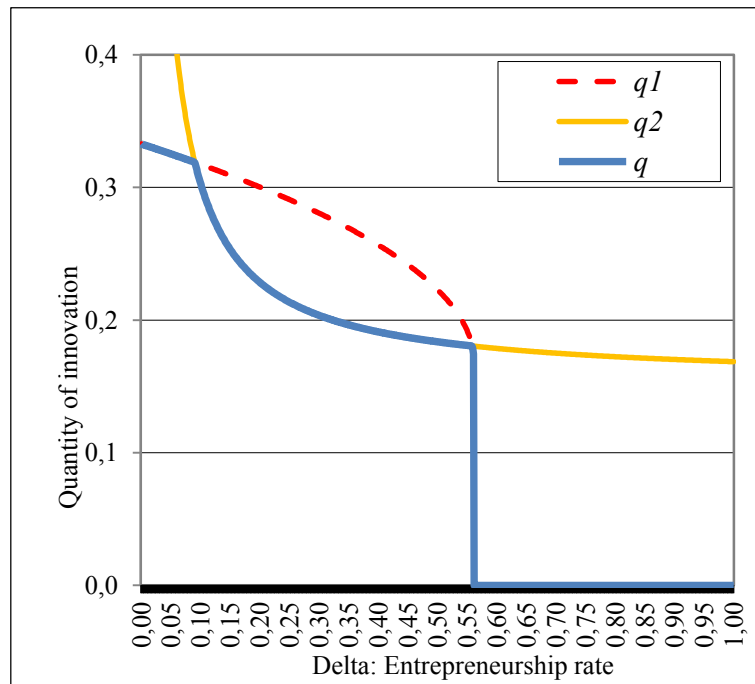


Figure 1. Solution of the (EP) model as a function of the propensity to innovation.

However, even in this simple case, it is not straightforward to calculate the amount of innovation in the economy, because it is very complex to integrate Q due to the existence (at least *a priori*) of non-linearities in the model and due to the shape of $f(\delta)$. For the specific case of the Uniform distribution, $U(0,1)$, the average of the quantity of innovation is $Q^*=0.1293$.

Concerning the growth of the economy, if the propensity to consume is $\mu = 0.8$ and the productivity is $\beta = 1.5$, the rate of innovation is $\gamma_y = 10.91\%$.

The allocation problem is also simulated both to validate the analytical solution and also to assess variability in terms of the number of entrepreneurs in the population.¹⁰ For $N=200$ entrepreneurs, 50 simulations are run. The main results are that the quantity of innovation in the economy, Q^* , lies between 21.2820 and 29.4426 with an average of 26.1124 (its theoretical value is $Q^* = 25.8557$); the rate of entrepreneurship, δ_0 , lies between 45% and 65.5% with an average of 56.54% (its theoretical value is $\delta_0 = 56.17\%$); and the rate of change of the economy, γ_y , lies between 10.9272% and 10.9277% (its theoretical value is $\gamma_y = 10.9276\%$).

		Entrepreneurship rate δ_0	Innovation quantity Q^*	Innovation rate of change γ
Theoretical	Average	56.17%	25.85	10.9076%
	Minimum	45.00%	21.28	10.9072%
Simulated	Average	56.54%	26.11	10.9076%
	Maximum	65.50%	29.44	10.9077%

Table 1. Results of the simulation.

6. Concluding remarks

In this paper we consider that entrepreneurial activities range from innovation to rent-seeking. This assumption allows us to link both innovation and rent-seeking in a unitary formal framework of entrepreneurship. From (EP) it is possible to establish the relationship between variables such as minimum profit threshold (a version of the satisfying behavior hypothesis (see Cooper, 2003: 26); opportunity of gains in other sectors of the economy (Baumol's structure of payoffs); the costs of innovation; and the quantity of innovation finally produced depending on the attitude of entrepreneurs themselves and society in general towards innovation effort -defined by $f(\delta)$ -, and the consequences of all this in the rate of growth of economic output.

In the allocation of effort (EP) problem, each entrepreneur δ compares the potential earnings associated with the production of the innovative good q^* , with an alternative source of earnings linked to rent-seeking activities, R . If the minimum condition to innovate is not satisfied, the entrepreneur chooses *not to introduce* new goods and engages in rent-seeking activities. The formal problem is an illustration of how the entrepreneur faces the dilemma of how to allocate the scarce resources to which he has

¹⁰ For a description of the pseudocode of the simulation model see Muñoz and Otamendi (2012).

access with the intention of getting what he esteems it should be a gain high enough linked to his activity.

The (EP) problem is solvable both analytically and by means of simulation techniques. In fact, simulation is a reliable option whenever analytical solutions are difficult to obtain. A validation exercise has been carried out using linear functions in order to provide an illustration of how the model works. A natural extension of (EP) would consist of parameterizing the input variables, specifically the entrepreneurial function $f(\delta)$, and performing a robust study of the behavior of the model with ad-hoc simulations.

The solution of (EP) establishes a relationship between the quantity of innovation for each entrepreneur as a function of the distance δ to pure innovation and rent-seeking. It also allows for the definition of an entrepreneurial index that quantifies the proportion (δ_0) of innovative entrepreneurs of a given economy. Entrepreneurs within the interval $0 \leq \delta \leq \delta_0$ will innovate and those with higher values of δ will not.

The implications on economic growth follow. Policy makers should implement policies that reduce the opportunity costs of innovation, R , as well as the entry barrier costs, α_0 ; the direct costs of production of innovations (for example subsidizing innovation activities), c ; and promoting an innovation culture that shapes the propensity to innovation distribution $f(\delta)$ in favor of innovative activities.

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APPENDIX I: REGIMES OF SOLUTION FOR (EP).

We assume that $u', v' > 0$ and $u'', v'' \leq 0$; $\pi'' < 0$; the marginal cost is positive $c > 0$; $\delta \in [0, 1]$; $R \geq \alpha_0 > 0$; the demand for innovative products is $p(0) > c$; and there exists a \hat{q} such that $p(q) = 0$, for all $q \geq \hat{q}$ and $p'(q) < 0$ for all $q < \hat{q}$; the properties of the threshold function are $\alpha'_\delta(\delta, \alpha_0, R) > 0$, $\alpha'_{\alpha_0}(\delta, \alpha_0, R) > 0$, $\alpha'_R(\delta, \alpha_0, R) > 0$; and the two extreme cases are $\alpha(0, \alpha_0, R) = \alpha_0$ and $\alpha(1, \alpha_0, R) = R$. From the assumptions on the demand for new goods, there exists a $\tilde{q} \in (0, \hat{q})$ such that $p(\tilde{q}) = c$. Therefore, $\pi(0) = \tilde{\pi}(\tilde{q}) = 0$. Finally, from the concavity of the profit function follows that there exists a global maximum of $\pi(q)$, $\bar{q} \in [0, \tilde{q}]$. Hence, it holds that $\pi'(q) > 0$ if $0 \leq q \leq \bar{q}$; and $\pi'(q) < 0$ when $\bar{q} \leq q \leq \tilde{q}$. This result -together with the function $\alpha(\bullet)$ - determines the feasible set for each δ type entrepreneur.

Regime	Entrepreneur	Binding restriction	Solution
A	Pure Innovator $\delta = 0$	$\pi(q) \geq \alpha_0$ Profit \geq Entry Barrier	$q^*(0) > 0$, if $\alpha_0 \leq \pi_{\max}$ $q^*(0) = 0$, if $\alpha_0 > \pi_{\max}$
B	Rent Seeker $\delta = 1$	$\pi(q) \geq R$ Profit \geq Other Rents	$q^*(1) > 0$, if $\alpha(1) \leq \pi(q^*(1))$ $q^*(1) = 0$, if $\alpha(1) = R > \pi(q^*(1))$
C	C1 Heterogeneous	$\alpha_0 \leq R \leq \pi_{\max}$ Profit \geq Other rents \geq Entry barrier	$q^*(\delta) > 0$; entrepreneurial activity for sure
	C2	$\alpha_0 \leq \pi_{\max} \leq R$ Profit in between rents and entry barriers	Some entrepreneurs allocate effort if propensity to innovate is high
	C3	$\pi_{\max} < \alpha_0$ Profit $<$ Entry barrier	$q^*(\delta) = 0$; No entrepreneurial activity at all

Table 2. Summary of results

From the functions $\pi(q)$ and $\alpha(\bullet)$, the feasible set for each type δ entrepreneur, B_δ , is determined by $B_\delta = \{q \in \mathbb{R} / \pi(q) \geq \alpha(\delta, \alpha_0, R)\}$. Figure 2 represents this set.

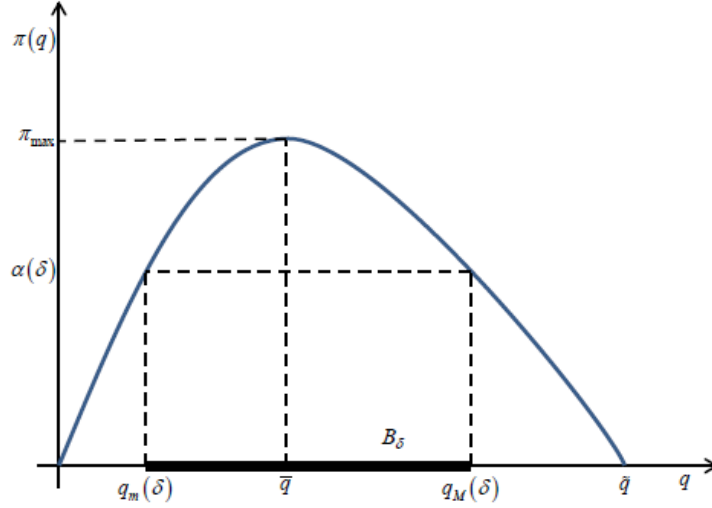


Figure 2. The set of feasible solutions B_δ .

For each value of $\pi(q)$ and $\alpha(\bullet)$ there are two possibilities:

$$\pi(q) \geq \alpha \begin{cases} \text{if } \alpha > \pi_{\max}, \text{ then } B_\delta = \emptyset \\ \text{if } \alpha \leq \pi_{\max}, \text{ then } B_\delta \neq \emptyset \end{cases} \quad (\text{AI.1})$$

In the second case, the feasible set is determined by the interval:

$$B_\delta = [q_m(\delta), q_M(\delta)] \quad (\text{AI.2})$$

The continuity of the utility function $U(\delta, q)$ and the fact that the feasible set is closed and bounded guarantees that there exists a global maximum (Weierstrass Theorem). Moreover, since $U(\delta, q)$ is strictly concave and the feasible set is convex, the Fundamental Theorem of Convex Programming guarantees that the global maximum of (EP) is unique. We denote this maximum by $q^*(\delta)$.

Innovation regimes when $B_\delta \neq \emptyset$

In the most interesting and general case there are two possibilities: (a) that the solution to (EP) is strictly interior, and (b) the solution lies on the frontier of the feasible set.¹¹ If the solution is *interior*, then it must hold that the optimal $q^*(\delta)$ is that which maximizes type δ entrepreneur's utility (for $\lambda = 0$); i.e.: a value of q such that $(1 - \delta)u'(q) + \delta v'(\pi(q))\pi'(q) = 0$. Equivalently, the condition that must be satisfied is:

$$\frac{v'(\pi(q))\pi'(q)}{u'(q)} = \left(\frac{1 - \delta}{\delta} \right) \quad (\text{AI.3})$$

¹¹ We apply the Kuhn-Tucker conditions.

In this condition the left-hand-side term may be interpreted as some type of marginal rate of substitution between innovations and profit that depends on the relative position of each entrepreneur in the distribution $f(\delta)$. On the other hand, the frontier solution applies when $\pi(q) > \alpha(\delta)$. In this second case, the solution is given by the restriction of (EP). In principle, this solution could be at either $q^* = q_m(\delta)$ or $q^* = q_M(\delta)$. From the condition $L'_q = 0$, and from the hypotheses on the functions of the problem, it holds that $\pi'(q^*) < 0$, and so the solution is given by $q^*(\delta) = q_M(\delta)$.

Another interesting feature of this model is that it generates a change in the solution regime when the solution passes from interior to binding or vice versa. This *asymmetric* behavior (some entrepreneurs behave like type 0s, while other like type 1s) is quite logical given the special functional form that has been assumed for the general utility function of the entrepreneurs, $U(\delta, q)$, i.e.: a linear combination of the two extreme, or pure, types of behavior. There is exactly one (or some) intermediate value that weighs the two parts of the utility function equally. (Appendix III provides the formal proof.)

Comparative statics

From Eq. (5), it is possible to analyze the relationships between the solution of the amount of innovation and the other arguments of the function. An unusual characteristic of (EP) solution is that it appears in *zones* within the distribution, which implies that these partial relationships within each zone must be considered, bearing in mind at all times the points at which one regimen of solution joins another. These joining points take place at the values $\hat{\delta}$ that satisfy the following condition:¹²

$$H(q_M(\delta)) < -\left(\frac{1-\delta}{\delta}\right) \quad (\text{AI.4})$$

In any case, it can be shown for both solution regimes (interior and frontier) that:¹³

$$\frac{\partial q}{\partial \alpha_0} < 0; \quad \frac{\partial q}{\partial R} < 0; \quad \frac{\partial q}{\partial c} < 0; \quad \frac{\partial q}{\partial p} > 0; \quad \frac{\partial q}{\partial \delta} < 0; \quad (\text{AI.5})$$

Consequently, given the functional relationships that have been assumed for demand, utility, variable costs, attitude towards innovation and the profit restriction, etc., the relationship $q^*(\delta) = q(\delta, \alpha_0, R, c, p)$ is *decreasing*¹⁴ in the position of the entrepreneur in the entrepreneurial capacity distribution, δ , in the economic cost of developing inventions, α_0 ; in the rents available within the economic system, R ; and in the variable production cost associated with innovations, c . Obviously, there is a positive dependence on the demand for innovative goods. That is, the closer is an entrepreneur (psychologically) to being of type 1 (i.e.: values profits relatively more), the greater is R ; the greater is the cost of production of innovations (a usual argument for subsidizing

¹² See Appendix II for a formal proof.

¹³ See Appendix III.

¹⁴ We say “decreasing” rather than the partial derivative is negative, since there exist points at which the functions are not differentiable. Since they are continuous, the form of the relationship is maintained.

innovation activities); and the greater the unitary production costs are; the lower the innovative activity of entrepreneurs as measured by q will be. The following proposition resumes the main results.

Proposition: There exists a single solution to (EP) for each δ , denoted by $q^*(\delta) = q^*(\delta, \alpha_0, R, c, p)$ that is continuous (except at the point $\tilde{\delta}$ whenever this is not greater than 1) on the interval $[0, 1]$, that is differentiable at almost all points. Moreover, the aggregate solution of (EP), $Q(\delta) = \int_0^1 q^*(\delta) f(\delta) d\delta$ is decreasing in α_0 , R and c , and increasing in f and p .¹⁵

APPENDIX II: SWITCHING POINT

Change in solution regime: the problem is to find out whether the solution of (EP) is interior or on the frontier (existence and uniqueness are guaranteed). Consider the function

$$H(q) = \frac{v'(\pi(q))\pi'(q)}{u'(q)} \quad (\text{AII.1})$$

We have $H(\bar{q}) = 0$ and H is strictly decreasing since $H'(q) < 0$. Therefore, the solution is interior if and only if

$$H(q_M(\delta)) < -\left(\frac{1-\delta}{\delta}\right) \quad (\text{AII.2})$$

In case (C.1) and in those cases of (C.2) in which the $B_\delta \neq \emptyset$ (see Table 2), these two innovation regimes are operative. Moreover, there exist some values $\hat{\delta}$ for which these regime changes will occur. For these $\hat{\delta}$ there will be an “inflection” (or connection) between the two types of solution. At these solutions it must hold that:

$$\left\{ \begin{array}{l} H(q_M(\delta)) = -\frac{1-\delta}{\delta} \\ q^*(\delta) = q_M(\delta) \end{array} \right. \text{ that is, } \hat{\delta} = \left\{ \begin{array}{l} \frac{v'(\pi(q^*(\delta)))\pi'(q^*(\delta))}{u(q^*(\delta))} = -\left(\frac{1-\delta}{\delta}\right) \\ \pi(q^*(\delta)) = \alpha(\delta) \end{array} \right. \quad (\text{AII.3})$$

APPENDIX III: COMPARATIVE STATICS

Consider $q^*(\delta) = q(\delta, \alpha_0, R, c, p)$. We have to distinguish between interior and frontier solutions.

1. Derivatives with respect to δ .
Interior solution.

¹⁵ For a proof of this proposition, see Appendixes III and IV.

$$\frac{v'(\pi(q^*(\delta)))\pi'(q^*(\delta))}{u(q^*(\delta))} = -\left(\frac{1-\delta}{\delta}\right) \quad (\text{AIII.1})$$

From which

$$q'(\delta) = \frac{u'(q(\delta)) - v'(\pi(q(\delta)))\pi'(q(\delta))}{\delta v''(\pi(q(\delta)))\pi'(q(\delta))^2 + \delta v'(\pi(q(\delta)))\pi''(q(\delta)) + (1-\delta)u''(q(\delta))} < 0 \quad (\text{AIII.2})$$

Frontier solution: we begin with $\pi(q(\delta)) = \alpha(\delta)$. Hence,

$$q'(\delta) = \frac{\alpha'(\delta)}{\pi'(q(\delta))} < 0 \quad (\text{AIII.3})$$

Note: we have discussed several solution regimes, which implies that one of them could be repeated. But in any case, the dependence is inverse (negative). On the other hand, it may well happen that, beginning with a particular value of δ the solutions “jump” to zero. We have not studied these cases since the solution is to always have zero innovations

2. Derivatives with respect to R.

Interior solution. Since the entrepreneur's restriction is constant in R for each interior value of δ , changes in R do not directly affect q^* in this case. There is, however, an indirect effect, since R determines the value of the function $\alpha(\bullet)$, and since $\alpha(\bullet)$ defines the set B_δ , changes in R will alter the values of δ that determine the changes in solution regime.

Frontier solution. In this case, R has a direct effect on the restriction through the function $\alpha(\bullet)$. Hence, since $\pi'(q(R))q'(R) = \alpha'(R)$, we have

$$q'(R) = \frac{\alpha'(R)}{\pi'(q(R))} < 0 \quad (\text{AIII.4})$$

3. Derivatives with respect to c.

Interior solution. Beginning with $(1-\delta)u'(q(c)) + \delta v'(\pi(c, q(c)))\pi'(c, q(c)) = 0$, taking

$\pi = \pi(c, q)$, $\pi' = \frac{\partial \pi}{\partial q}$ and since $\frac{\partial \pi}{\partial c} = q$, together with the fact that

$p''(q(c))q(c) + 2p'(q(c)) = \pi''(q(c))$, we may conclude that

$$q'(c) = \frac{q\delta v''(\pi(c, q(c)))\pi'(c, q(c)) + \delta v'(\pi(c, q(c)))}{\delta v''(\pi(c, q(c)))\pi'(c, q(c))^2 + \delta v'(\pi(c, q(c)))\pi''(c, q(c)) + (1-\delta)u''(q(c))} < 0$$

(AIII.5)

Frontier solution for $\pi = \pi(c, q(c)) = \alpha(\bullet)$

$$q'(c) = \frac{q}{\pi'(c, q(c))} < 0 \quad (\text{AIII.6})$$

4. Dependence on p.

An additional problem is that of the dependence of q on changes in the demand function, $p(q)$. In this case the dependence is with respect to a function, which makes it impossible to evaluate, although it is clear that greater demand will imply greater profit opportunities and, additionally, greater incentives to produce greater amounts of innovative goods.

APPENDIX IV: STOCHASTIC DOMINANCE

More interesting is to show how the solution to (EP) is affected by changes in the aggregate distribution of entrepreneurial capacity. Although the solution is not trivial -since it depends on a characteristic of the distribution functions with very little a priori information -, it is possible to establish one important result: the average level of innovation in the economy -as measured from (9)- is greater in those economies whose *entrepreneurial capacity* distribution function first order stochastically dominates another. That is, given two different entrepreneurial capacity density functions, f_1 and f_2 , where we define $F_i(\delta) = \int_0^\delta f_i(s)ds$, for $i = 1, 2$, if it holds that

$$F_1(\delta) \geq F_2(\delta), \forall \delta \in [0, 1] \quad (\text{AIV.1})$$

then we have $Q_1 \geq Q_2$, for given values of the other variables.¹⁶ Graphically, distribution f_1 first order stochastically dominates f_2 if their relative positions are as follows:

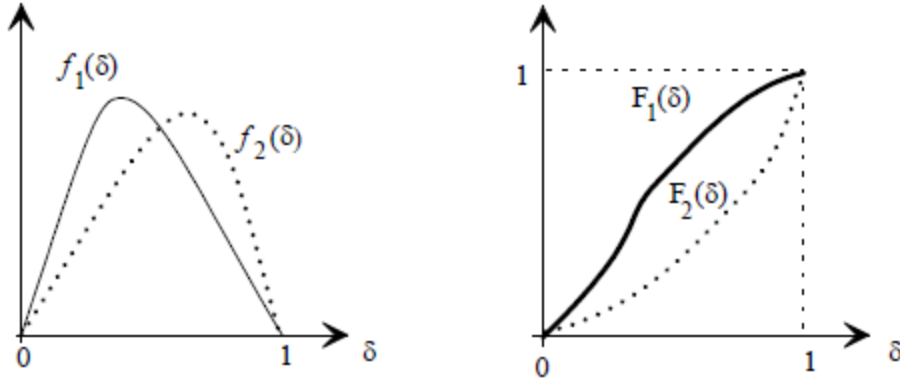


Figure 2.

If we denote $Q_1 - Q_2 = \int_0^1 q(\delta)(f_1(\delta) - f_2(\delta))d\delta$ and integrate by parts, then

$$Q_1 - Q_2 = q(\delta)(F_1(\delta) - F_2(\delta))\Big|_0^1 - \int_0^1 q'(\delta)(F_1(\delta) - F_2(\delta))d\delta = -\int_0^1 q'(\delta)(F_1(\delta) - F_2(\delta))d\delta > 0 \quad (\text{AIV.2})$$

we have that $Q_1 \geq Q_2$.

APPENDIX V: GROWTH MODEL

We assume the following definition of capital:

¹⁶ See Appendix IV.

$$K(t) = A(t)Q(t) \quad (\text{AV.1})$$

where $K(t)$ is the accumulated amount of goods that have been incorporated in the production process, and $A(t)$ is the number of varieties of new production goods that have been generated up to the moment of time t . That is to say, at each moment t , the amount of varieties that can be used in the production of output is given by the amount (and number) of innovative goods of previous periods, as well as those that are introduced at moment t . Hence, the introduction of new goods will imply an increase (change) in $A(t)$ equal to $\dot{A}(t)$; and $A(t)$ will depend on both the dynamics of inventions and the dynamics of innovations.

In fact:

$$\gamma_y = \gamma_A = \frac{\dot{A}}{A} = \frac{(1-\mu)\beta Q - \dot{Q}c}{QC - \alpha_0} \quad (\text{AV.2})$$

In the Rivera-Batiz and Romer model, changes in A are denoted as: $\dot{A} = \tau HA$; that is, the rate of growth of A is proportional to the human capital in the system, H , and a parameter that measures the productivity of this capital, τ . In these types of model, it also turns out that all inventions are introduced into the economic system as innovations. In our case, things do not happen with such a high degree of automation, since the entrepreneurs, from the problem (EP) and their relative types, δ , will determine the amount of each invention (variety) that will be produced, where a possible solution is that no positive amount at all is produced, i.e.: an invention is not transformed into an innovation.